A multiphase lattice Boltzmann model with sharp interfaces

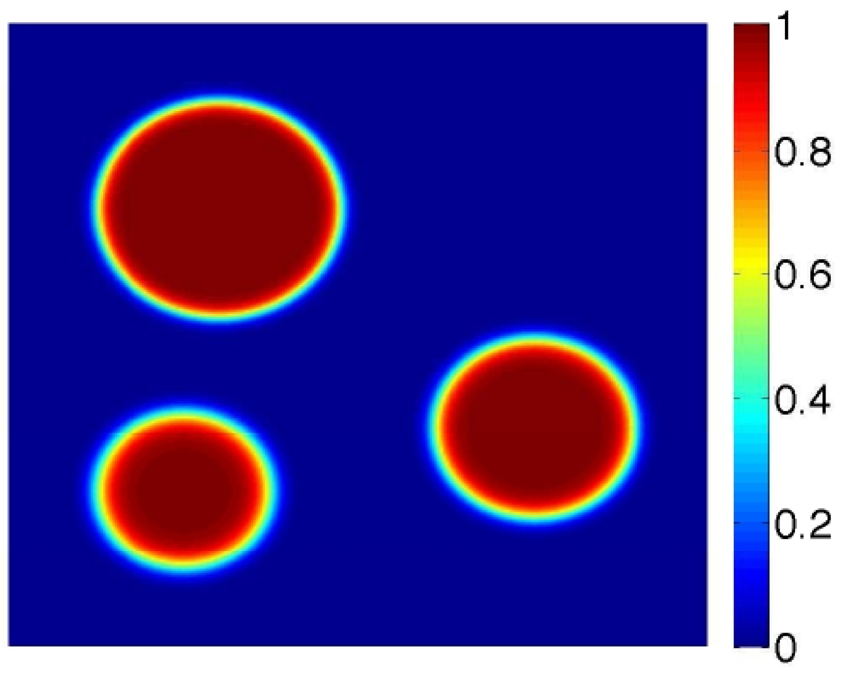
Tim Reis
Centre for Mathematical Sciences, Plymouth University

Introduction and motivation

The multiphase lattice Boltzmann method (LBM) is a numerical tool for simulating fluid flow. Its potential for high–end computing has made it an attractive algorithm for solving immiscible flow problems, where two fluids are separated by a vanishingly thin interface. Unfortunately, multiphase LBMs suffer from "lattice pinning"; a spurious phenomenon where the interface becomes fixed to the computational grid. We wish to understand and resolve this shortcoming.

The phase field

We introduce a phase field ϕ to represent the different fluids. Regions where $\phi \approx 0$ represent one phase and regions where $\phi \approx 1$ the other phase.



In principle, ϕ advects with the fluid velocity $\mathbf{v}$,

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = 0,$$

but in simulations numerical diffusion causes the interface between fluids to become smeared over too many gridpoints.

Interface sharpening

To counteract numerical diffusion and maintain narrow phase boundaries we add a "sharpening term" [1] to the hyperbolic transport equation:

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = S(\phi) = -\frac{1}{T} \phi \,(1 - \phi)\,(1/2 - \phi).$$

The sharpening term $S(\phi)$ has an unstable critical point at the threshold value of $1/2$ and drives $\phi \to \{0, 1\}$ at each time increment:

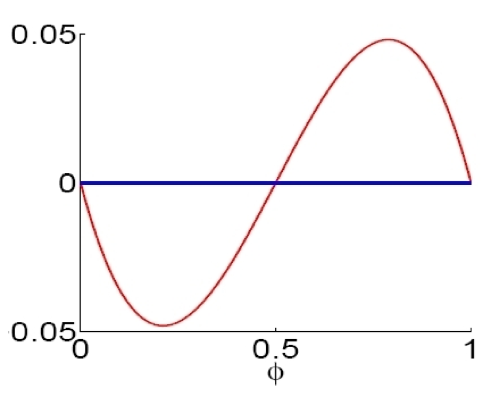


Fig. 1: Plot of $S(\phi)$

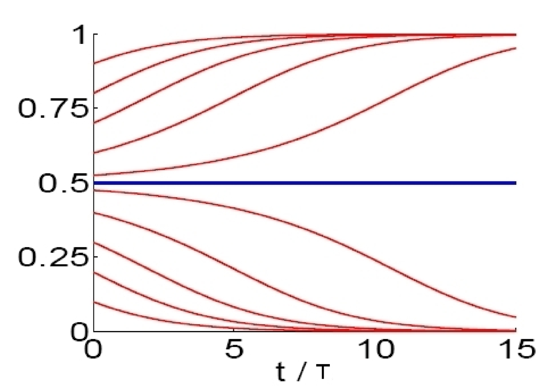
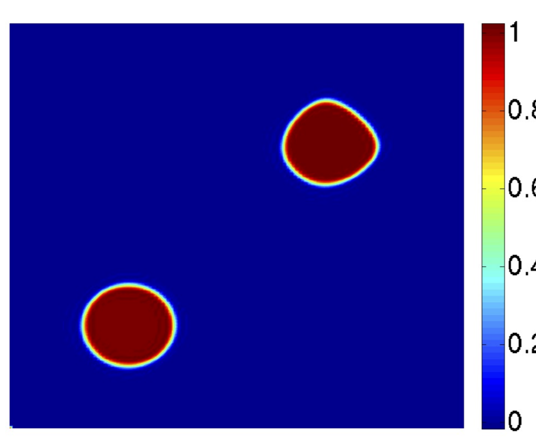


Fig. 2: Plot of $\phi_t = T^{-1}S(\phi)$.

Stiffness and interface pinning

The sharpening equation is numerically "stiff" when the timescale T of the source term is much shorter than a typical advective timescale.



Stiffness causes the interface to become "pinned" [2].

Multiphase lattice Boltzmann model

We introduce two lattice Boltzmann equations with distribution functions $f_i(\mathbf{x}, t)$ and $g_i(\mathbf{x}, t)$ for the fluid flow and phase field, respectively, where subscript i refers to the veclocity $\mathbf{c}_i$ on the lattice (Fig. 3):

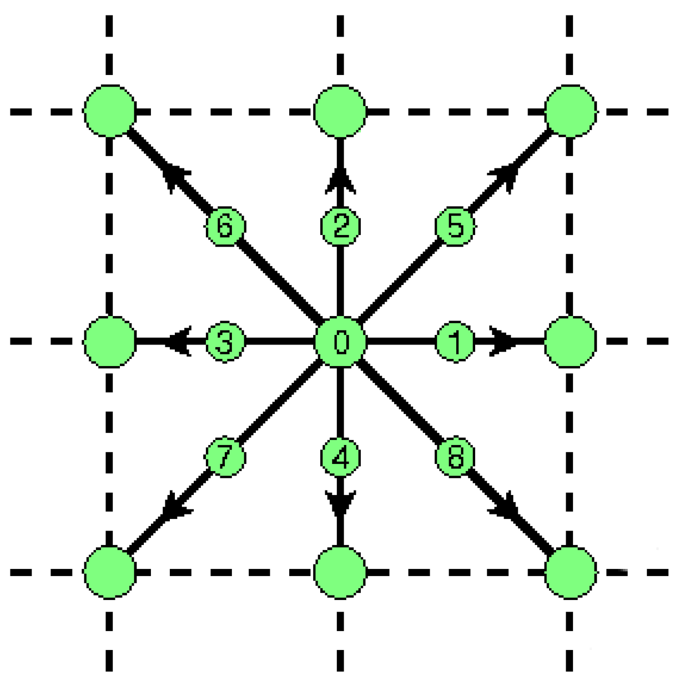


Fig. 3: Lattice with velocities $\mathbf{c}_i, i = 0, \ldots, 8$

$$\frac{\partial f_i}{\partial t} + \mathbf{c}_i \cdot \nabla f_i = -\frac{1}{\tau_f}\left(f_i - f_i^{(e)}\right),$$

$$\frac{\partial g_i}{\partial t} + \mathbf{c}_i \cdot \nabla g_i = -\frac{1}{\tau_g}\left(g_i - g_i^{(e)}\right) + R_i,$$

subject to the constraints

$$\sum_i f_i = \sum_i f_i^{(e)} = \rho, \quad \sum_i f_i \mathbf{c}_i = \sum_i f_i^{(e)} \mathbf{c}_i = \rho \mathbf{v},$$

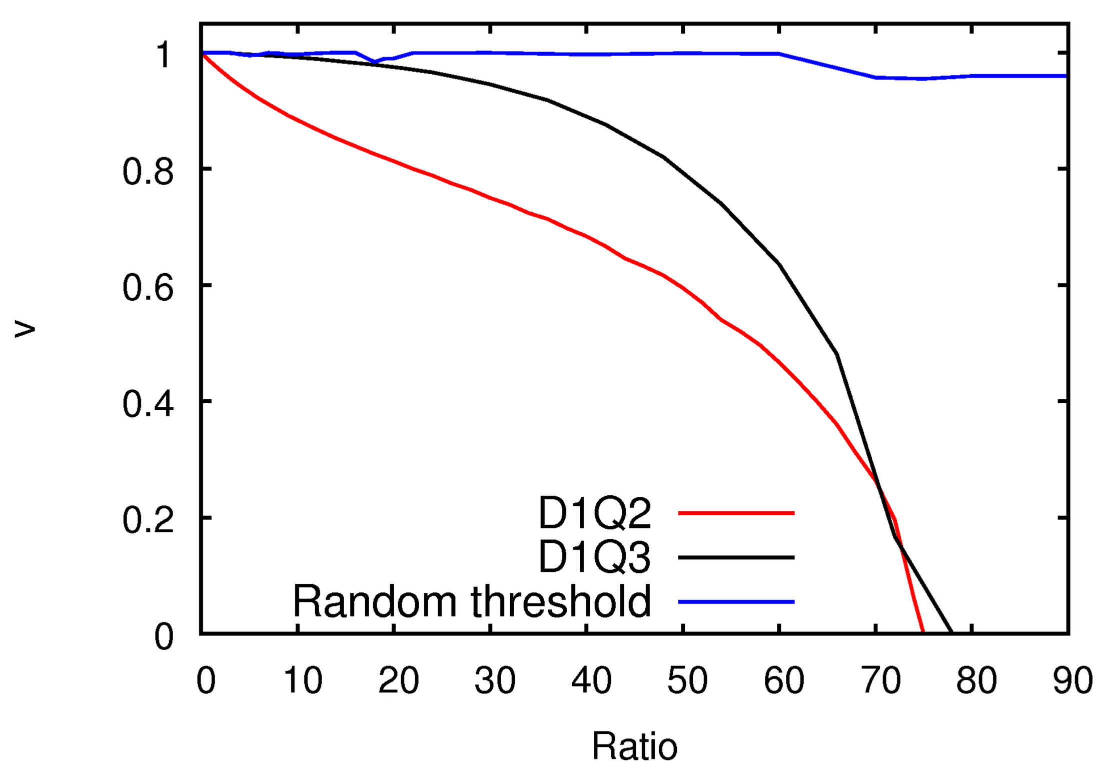
$$\sum_i f_i^{(e)} \mathbf{c}_i \mathbf{c}_i = \frac{\rho}{3} + \rho \mathbf{v}\mathbf{v} + \sigma\left(\mathbf{I} - \mathbf{n}\mathbf{n}\right),$$

$$\sum_i g_i = \sum_i g_i^{(e)} = \phi, \quad \sum_i g_i^{(e)} \mathbf{c}_i = \phi \mathbf{v}, \quad \sum_i R_i = S(\phi).$$

Here, the τ control diffusion, ρ is the fluid density, σ is the surface tension, and $\mathbf{n} = \nabla\phi / |\nabla\phi|$ is the interface unit normal.

Sharpening and pinning

The above model furnishes in the macroscopic limit the continuous surface force equations for multiphase flow coupled to the interface sharpening equation [2]. However, it suffers from pinning.



Replacing the threshold $1/2$ in $S(\phi)$ with a uniformly distributed quasi-random variable greatly reduces pinning and allows narrow interfaces to advect with the fluid [2, 3].

This figure plots the interface propagation speed as a function of the ratio of timescales using a constant threshold (D1Q2 and D1Q3; referring to different discretisations) and a random threshold. High ratio implies greater stiffness and narrow interfaces. The constant threshold causes the interface to deviate significantly from the fluid velocity (scaled to unity), eventually becoming completely pinned. The random threshold model allows very narrow interfaces to advect correctly.

Numerical validation

The model accurately computes layered Poiseuille flow of two fluids with different viscosities and is able to maintain an interface of minimal width (one gridpoint).

Laplace's law is also confirmed numerically and shown to be free of numerical error in the surface tension.

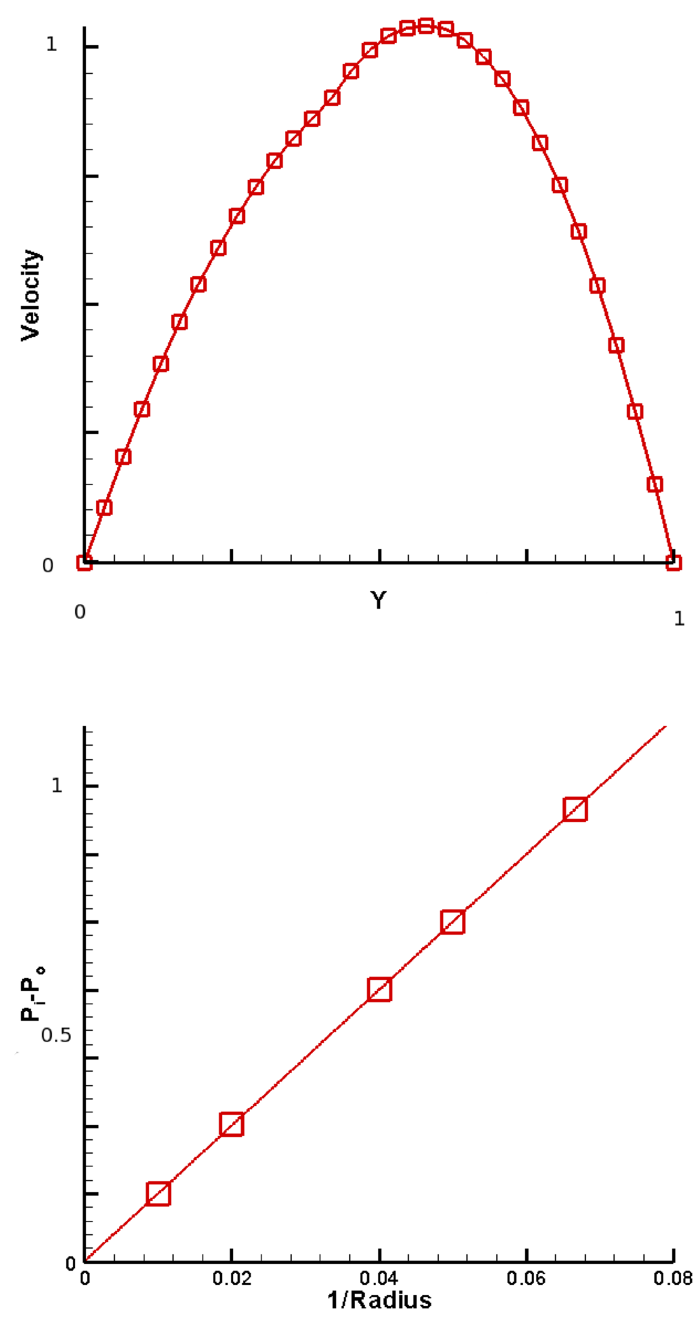


Fig. 4: (Top) Layered Poiseuille flow with boundaries at $Y = 0$ and $Y = 1$. The symbols are the numerical predictions and the solid line is the analytical solution. (Bottom) Laplace's law. The slope of the curve is the surface tension σ. P_i and P_o are the scaled pressures inside and outside a bubble.

Viscous fingering

The model can be used to simulate a less viscous fluid (red) penetrating a more viscous fluid (blue), as is often encountered in carbon sequestration and enhanced hydrocarbon recovery applications. The unstable interfaces causes fingering phenomena.

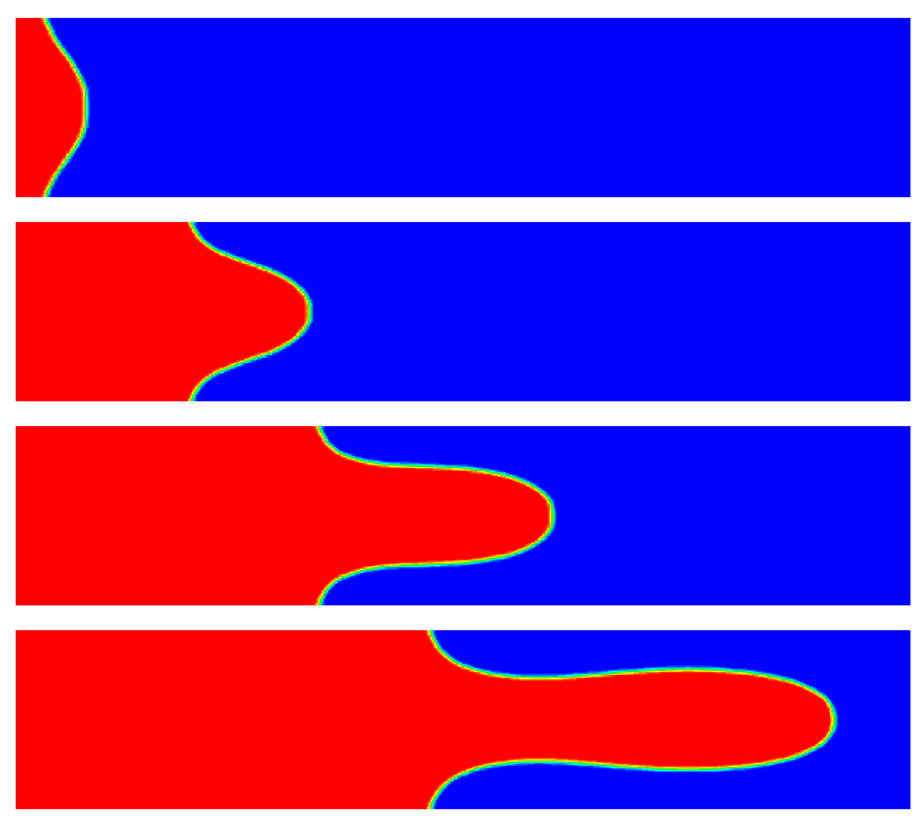


Fig. 5: Numerical simulation of viscous fingering. The capillary number is $Ca = 0.1$ and the viscosity ratio is 20.

Future Work

We plan to use this model to study industrially and environmentally relevant flows including carbon sequestration and transport in shales.

References

[1] R. J. LeVeque and H. C. Yee, J. Comput. Phys., 86 (1990), 187-210
[2] T. Reis and P. J. Dellar, Comp. Fluids, 46 (2011), 417-421
[3] W. Bao and S. Jin, J. Comput. Phys., 163 (2000), 216-248