Probing the quantum vacuum with intense lasers

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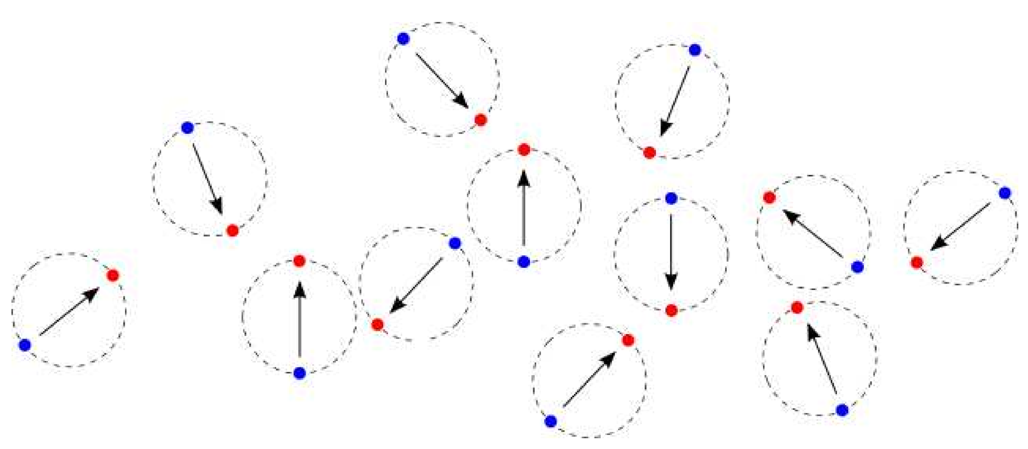
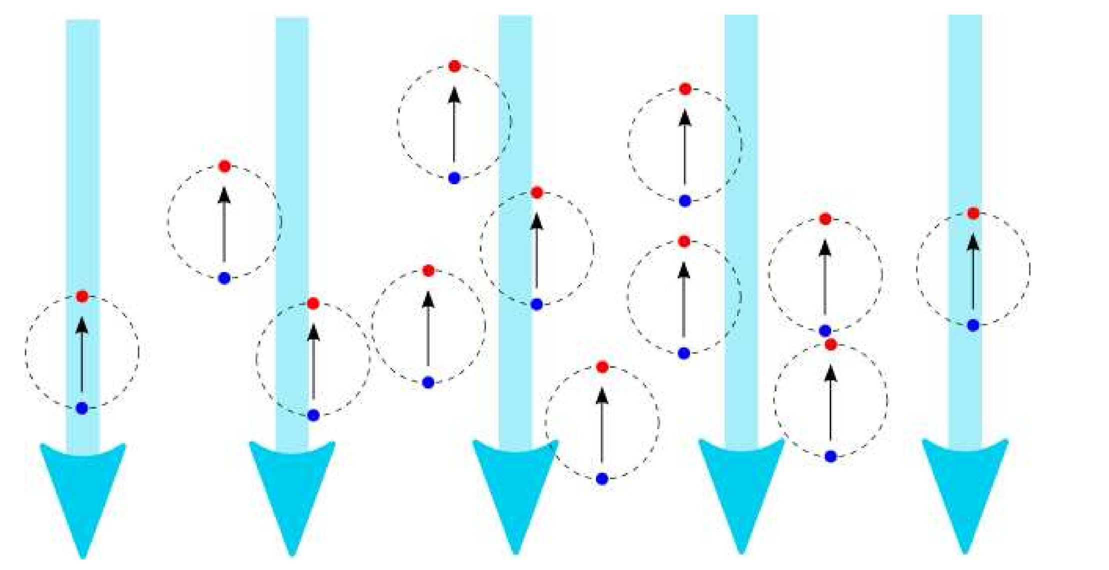
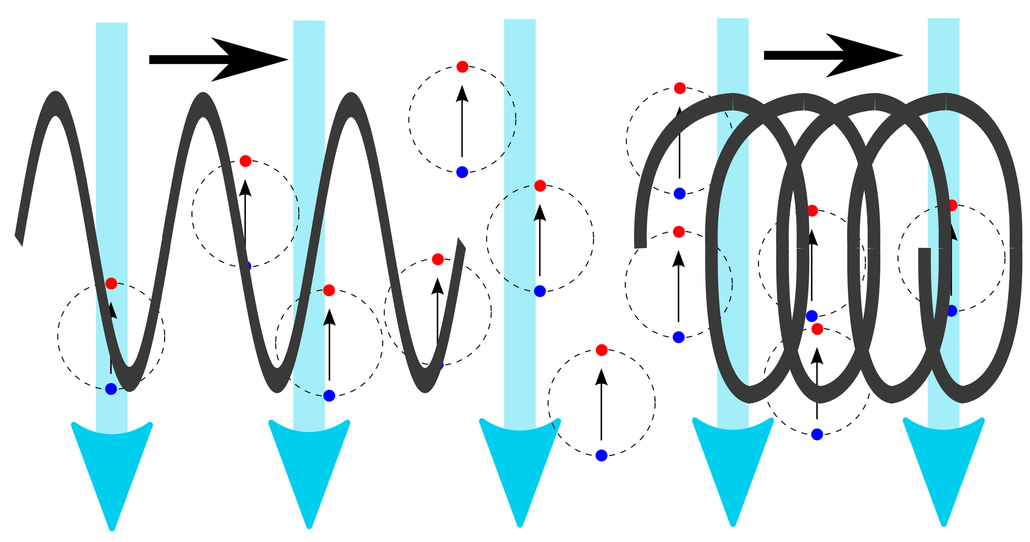
Polarising the quantum states of the vacuum





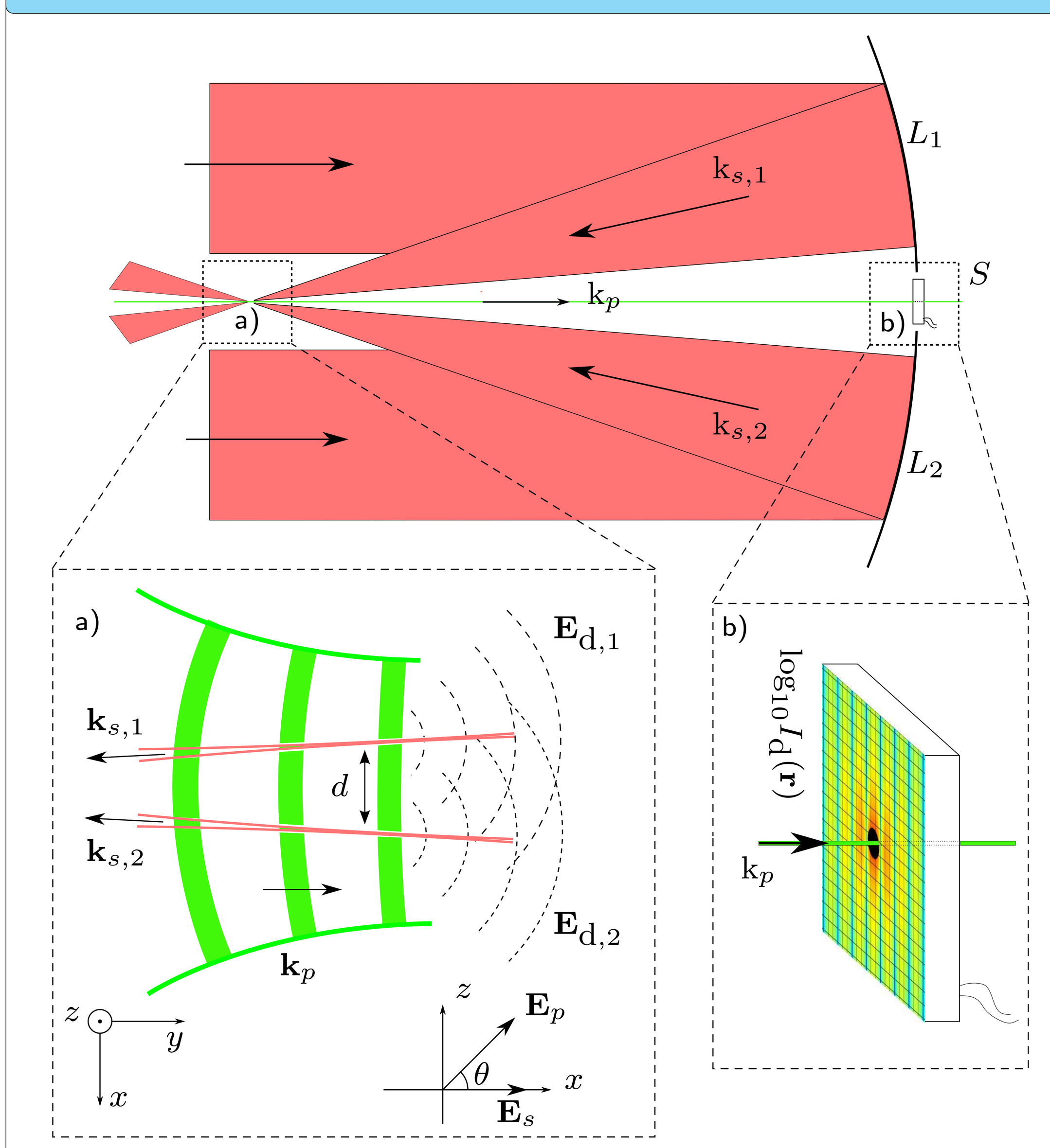


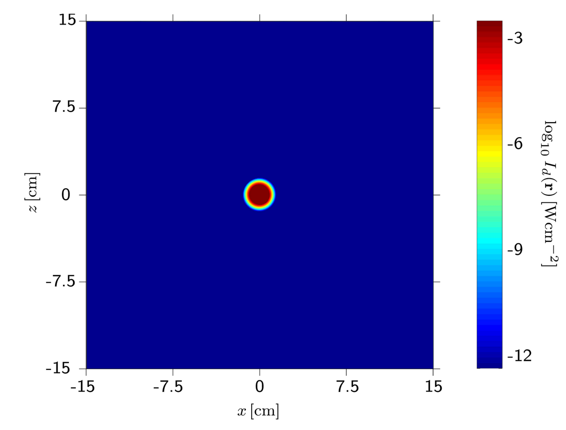
Quantum mechanics predicts the existence of so-called "virtual" states that come into and out of existence on very short timescales. These virtual states are considered to be part of the "quantum vacuum". A useful analogy is to consider the vacuum to be populated by virtual particle pairs that live on a space and time scale given by the Heisenberg uncertainty relation $\Delta\mathcal{E}\Delta t \approx \hbar$, where $\Delta\mathcal{E}$ is the uncertainty in energy, Δt a time interval of measurement and $\hbar = 6.626 \times 10^{-34}$ Js Planck's constant.

Einstein's famous equation $\mathcal{E} = mc^2$ gives a size for $\Delta\mathcal{E}$ in the uncertainty relation, we see that the lighter a particle, the longer it "exists" for. Virtual electrons and positrons – the lightest known charged elementary particles – are expected to be the most easily detectable. They occur in pairs, because electric charge is conserved in Nature. Since these pairs are charged, they will be *polarised* when exposed to an intense electromagnetic (EM) field (such as produced by a laser) and generate their own EM field that *opposes* the laser's.

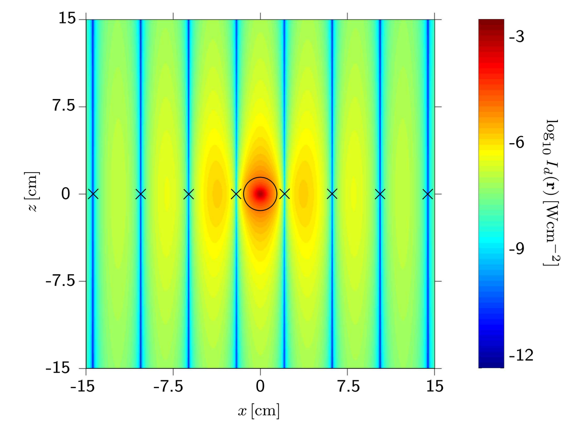
When a second laser pulse passes through a region of the vacuum with polarised virtual pairs, it can be altered in a number of ways. In this picture, the probe initially oscillates linearly but then oscillates elliptically, gaining *helicity*, which can be measured as a signal of the quantum vacuum interaction [1].

Example experimental set-up to measure real photon-photon scattering





Scattered probe photons (EM field) *without* vacuum interaction.



Scattered probe photons (EM field) *with* vacuum interaction.

In the figure, two very intense laser pulses (red) act to polarise the vacuum. The probe pulse (green) is then "scattered" on the two, slit-like polarised regions, which emit photons that interfere and produce a *diffraction pattern* [2]. In terms of the electric field, $\mathbf{E}(t,\mathbf{r})$, one solves the inhomogeneous wave equation (a hyperbolic partial differential equation, coupled with the magnetic field wave equation):

$$\frac{\partial^2\mathbf{E}(t,\mathbf{r})}{\partial t^2} - \boldsymbol{\nabla}^2\mathbf{E}(t,\mathbf{r}) = \frac{\partial\mathbf{J}(t,\mathbf{r})}{\partial t}; \qquad \mathbf{J}(t,\mathbf{r}) \sim \frac{\partial}{\partial\xi}E^3(t,\mathbf{r})$$

where $\mathbf{J}(t,\mathbf{r})$ is the *current* due to the polarised virtual pairs, which occurs in Maxwell's equations of electrodynamics, ξ is some length or time co-ordinate and the right-hand equation is an indicative form of the current. This equation is *non-linear* because of the form of the current, or inhomogeneity. Since the scattered field $\Delta\mathbf{E}$ is much smaller than the probe field $\mathbf{E}_p$, and since the inhomogeneity is much smaller than the fields of the probe and the "strong", polarising pulse $\mathbf{E}_s$, one can solve this wave equation using its Green's function $G(t,|\mathbf{r}'|)$:

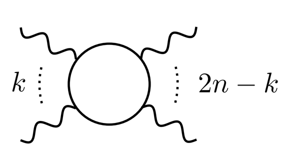
$$\Delta\mathbf{E}(t,\mathbf{r}) = \int dt' d^3\mathbf{r}'\, G(t-t',|\mathbf{r}-\mathbf{r}'|)\,\frac{\partial\mathbf{J}(t',\mathbf{r}')}{\partial t'}; \quad G(t,|\mathbf{r}|) = \frac{\theta(t)\delta(t-|\mathbf{r}|)}{4\pi|\mathbf{r}|}$$

assuming that $J \sim E_s^2\partial E_p/\partial\xi$, and $\mathbf{E}_d \approx \mathbf{E}_p + \Delta\mathbf{E}$ is the total probe field "diffracted" from interaction with the strong field on the virtual pairs.

Quantum Processes

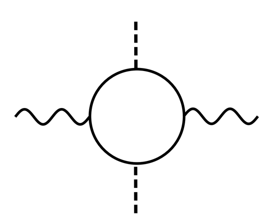
The interaction of EM fields and virtual pairs is described by the part of the *standard model* of particle physics called *quantum electrodynamics* (QED).

In a useful diagrammatic language, photons are represented as wavy lines with electrons and positrons as solid lines. In this language, laser pulses interact with virtual pairs via *real photon-photon scattering*, which, by charge-conjugation symmetry must involve even numbers of photons.

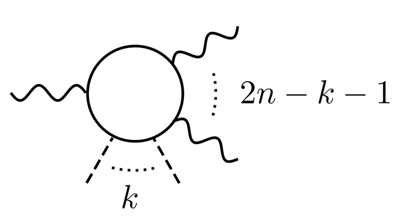


$k, n \in \mathbb{N}$

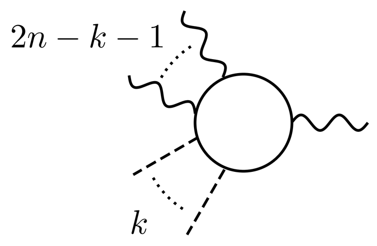
It is useful to split the EM (photon) field into parts produced by the probe $\mathbf{E}_p$ (wavy lines) and strong $\mathbf{E}_s$ (dashed lines) laser pulses, where $E_p \ll E_s$. Multiple *channels* of probe photon scattering can then be identified [4]:



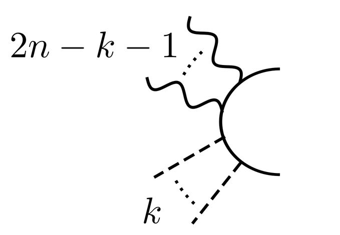
Four-photon scattering (currently searched for in experiments [5]).



Photon splitting (parametric frequency down-conversion).



Photon merging (parametric frequency up-conversion / high harmonic generation).



Pair creation (light into matter conversion into real electron-positron pairs).

References

[1] T. Heinzl et al., *On the observation of vacuum birefringence*, Opt. Commun. **267** (2006) 318–321.

[2] B. King and A. Di Piazza and C. H. Keitel, *A matterless double-slit*, Nature Photon. **4**, 92–94 (2010)

[3] G. Dunne from M. A. Shifman, A. Vainshtein and J. Wheater, *From fields to strings: Circumnavigating Theoretical Physics*, World Scientific (2005)

[4] B. King and T. Heinzl, *Measuring Vacuum Polarisation with High Power Lasers*, High Power Laser Science and Engineering, **4**, e5 (2015)

[5] H. Schlenvoigt, T. Heinzl, U. Schramm, T. Cowan, and R. Sauerbrey, *Detecting vacuum birefringence with X-ray free electron lasers and high-power optical lasers: A feasibility study*, Phys. Scripta **91**, 023010 (2015).

Where are the virtual pairs?

Why can we describe the scattering of photons in the probe laser pulse with photons from the polarising pulse, without referring to the virtual pairs? This is a success of an *effective field theory*. Suppose the field content comprises "light" ϕ and "heavy" Φ fields. An arbitrary transition amplitude can be written as a functional integral over these fields, weighted in the usual fashion by the action $S[\phi,\Phi]$. If we begin and end with the vacuum and are only interested in the dynamics of the lighter fields ϕ, we can write this amplitude as an integral over these fields, weighted by an *effective action*, $W[\phi]$, in the following way, in which the heavier degrees of freedom are "integrated out" ($|0\rangle$ a vacuum state)

$$\langle 0|0\rangle = \int \mathcal{D}\phi\,\mathcal{D}\Phi\;\mathrm{e}^{i\,S[\phi,\Phi]} = \int \mathcal{D}\phi\;\mathrm{e}^{i\,W[\phi]}.$$

Following this procedure for QED, the effective action can be written as the *functional determinant* over the gauge field A_μ:

$$W[A] = i\mathrm{Tr}\ln\left[\frac{\gamma^\mu\left(i\partial_\mu + eA_\mu\right) + m}{\gamma^\mu\partial_\mu + m}\right].$$

If the EM field is *slowly-varying* on length scales much greater than the electron Compton wavelength, the EM fields can be approximated as constant and the corresponding *effective Lagrangian* – the Heisenberg-Euler Lagrangian – can be written:

$$\mathcal{L}_{\mathrm{HE}} = -\frac{m^4}{8\pi^2}\int_0^\infty ds\,\frac{\mathrm{e}^{-s}}{s^3}\left[s^2 ab\cot as\coth bs - 1 + \frac{s^2}{3}(a^2-b^2)\right]$$

where the EM fields are in the scaled EM invariants $\mathcal{F} = (E^2-B^2)/2E_{\mathrm{cr}}^2$, $\mathcal{G} = \mathbf{E}\cdot\mathbf{B}/E_{\mathrm{cr}}^2$ in the secular invariants $a,b = [\sqrt{\mathcal{F}^2+\mathcal{G}^2} \pm \mathcal{F}]^{1/2}$. The Sauter-Schwinger limit E_{cr} is the strength of electric field required to do work over the electron Compton wavelength equal to the electron rest mass and is the natural QED field scale. $\mathcal{L}_{\mathrm{HE}}$ is a remarkable object and has been often studied. For "magnetic backgrounds" $B > E$, the weak-field ($a,b \ll 1$) perturbative expansion is *divergent* but Borel resummable, for "electric backgrounds" $E > B$, the expansion is *divergent*, non Borel-resummable and contains an imaginary *non-perturbative* part, which is a signature of *vacuum instability* [3].