Very Special Relativity as a background field theory

Anton Ilderton

Plymouth University, UK

2016-10-05

RESEARCH & INNOVATION
Marie Skłodowska-Curie actions

Stiftelsen Olle Engkvist Byggmästare
Signhild Engkvists Stiftelse

- 1605.04967 [hep-th], Phys. Rev. **D** 94 (2016) 045019

Outline

1. Intro: Lorentz violation and Very Special Relativity
2. Intro: Strong Field QED.
3. Particle motion in VSR and in background fields
4. SF-QED $\leftrightarrow$ SIM(2)-QED correspondence

What is VSR?

- New physics from violation of spacetime symmetries?

Liberati, Class. Quant. Grav. 30 (2013) 133001

VSR

1. Replace Lorentz group (6 dim) → subgroup (2,3,4 dim.)
2. Keep translation invariance.
3. Build (Lorentz invariance violating) QFTs.

Cohen & Glashow, PRL 97 (2006) 021601

- Original motivation: allows for neutrino masses.

Cohen & Glashow, hep-ph/0605036

SIM(2)

- Largest subgroup: 4-dimensional SIM(2).

$$J_3, \qquad K_3, \qquad T_1 = K^1 + J^2, \qquad T_2 = K^2 - J^1$$

- c constant, same dispersion. $\rightarrow$ p^2 = constant.
- SIM(2) + parity = Lorentz. $\rightarrow$ Parity violation.

  B. Bucher et al, PRL 116 (2016) 112503
- Null hyperplanes invariant... $\rightarrow$ Lightlike stability group
- Preferred lightlike direction $\rightarrow$ From an æther?

  $n_\mu = (1, 0, 0, 1)$

  Gibbons et al, PRD 76 (2007) 081701
  Cheon et al, PLB 679 (2009) 73

  Direction, but no velocity

SIM(2) and mass terms

- SIM(2) permits nonlocal terms of the form $\frac{n_\mu}{n.\partial}$

Dirac equation. . . . . . in SIM(2)

$$\not{p} = m \quad \longrightarrow \quad \not{p} - \frac{\delta m^2}{2n.p}\not{n} = m$$

Mass-shell:

$$p^2 = m^2 \quad \longrightarrow \quad p^2 = m^2 + \delta m^2 \; .$$

- $\delta m \leftrightarrow$ small neutrino mass. Cohen & Glashow PRL 97 (2006) 021601

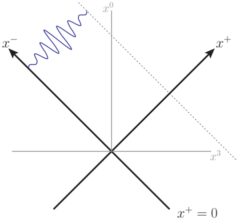
Particle motion in background fields

- Background plane wave field (basic laser model):

$$eA_\mu(x) =: a_\mu(n.x) = ma_0(0, \cos(\omega n.x), \sin(\omega n.x), 0)$$
$$n_\mu = (1, 0, 0, 1)$$

- Lightlike wave vector n_μ.
- Frequency ω.
- Intensity (coupling) a_0.
- Integrable model

(Lorentz, KG, Dirac all solvable)



Sengupta, Bull. Math. Soc. Calcutta 44 (1952) 175, Volkov, Z. Phys. 94 (1953) 250

Classical motion

- σ, rates involve averages over many field oscillations.

Nikishov & Ritus 1963

- Example: classical momentum.

$$\pi_\mu(n.x) = p_\mu - a_\mu(n.x) + \frac{2p.a(n.x) - a^2(n.x)}{2n.p} n_\mu$$

average $\downarrow$ (linear in $a \to 0$, quadratic $\not\to 0$)

$$q_\mu := \langle \pi_\mu \rangle = p_\mu + \frac{a_0^2 m^2}{2n.p} n_\mu \ .$$

VSR connection

$$\pi_\mu \longrightarrow q_\mu = p_\mu + \frac{a_0^2 m^2}{2n.p} n_\mu \qquad \text{quasimomentum}$$

$$\pi^2 = m^2 \longrightarrow q^2 = m^2 + a_0^2 m^2 \qquad \text{effective mass}$$

T.W.B. Kibble, Phys. Rev. **138** (1965) B740,

Harvey, Heinzl, Ilderton, Marklund PRL 109 (2012) 100402

- Frequency scale drops out
- Averaged motion in plane waves described by VSR?

$$a_0^2 m^2 \leftrightarrow \delta m^2$$

QED in background fields

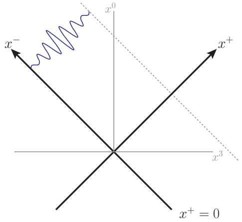
- QED in a background field / BSM photon background.

$$
\begin{aligned}
eA_\mu(x) =: a_\mu(n.x) &= ma_0\big(0, \cos(\omega n.x), \sin(\omega n.x), 0\big) \\
n_\mu &= (1, 0, 0, 1)
\end{aligned}
$$

⋆ Very high frequency scale ω.

⋆ Cannot be probed by typical processes.

⋆ Effective physical observables: averages.



Averaging

Basic idea.

- Obtain an "effective" theory by:

1. Killing rapidly oscillating terms — e.g. $a_\mu(n.x)$
2. Keeping slowly varying terms — e.g. $a^2(n.x) = -\delta m^2$

- Classically: gives quasi-momentum/VSR momentum.
- Quantum: consistent, gauge invariant truncation of QED?

Dirac equation

- Dirac equation in the background plane wave:

$$(i\partial\!\!\!/ - a\!\!\!/)\psi - m\psi = 0$$

- Volkov solutions:

$$\psi = \int \mathrm{d}p \left(1 + \frac{n\!\!\!/ a\!\!\!/(n.x)}{2n.p}\right) u_p \exp\left[-ip.x - i\int^{n.x} 2p.a - a^2\right] b_p + \ldots$$

- Average: $\psi \to \psi_{\mathsf{av}}$. The averaged field obeys: $\boxed{a_0^2 m^2 \leftrightarrow \delta m^2}$

$$\left(i\partial\!\!\!/ - \frac{\delta m^2}{2i n.\partial} n\!\!\!/\right)\psi_{\mathsf{av}} - m\psi_{\mathsf{av}} = 0$$

- The Dirac equation in SIM(2) VSR!

Averaging in QED

- Fermion propagator in a plane wave. D.M. Volkov, Z. Phys. 94 (1935) 250

H. Mitter, Acta Phys. Austr. Suppl. 14 (1975) 397

$$S_{\text{Volk}}(x,y) = \int \mathrm{d}p\, e^{-i\ldots}\left(1+\frac{\not{n}\not{a}(n.x)}{2n.p}\right)\frac{\not{p}+m}{p^2-m^2+i\epsilon}\left(1+\frac{\not{a}\not{n}(n.y)}{2n.p}\right)$$

Averaging in QED

- Fermion propagator in a plane wave. D.M. Volkov, Z. Phys. 94 (1935) 250

  H. Mitter, Acta Phys. Austr. Suppl. 14 (1975) 397

$$S_{\mathrm{Volk}}(x,y) \xrightarrow{\text{average}} S_{\mathrm{vsr}}(x-y) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{\not{q} + m - \frac{\delta m^2 \not{n}}{2n.q}}{q^2 - m^2 - \delta m^2 + i\epsilon} e^{-iq.(x-y)}$$

- Restores translation invariance, as in VSR. ✓

  Cohen & Glashow PRL 97 (2006) 021601

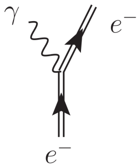
- Poles shifted to VSR mass. ✓

- Spin–$\frac{1}{2}$ propagator of VSR. ✓

The VSR vertex and the Ward Identity

☹ However . . .

- Extra slowly-varying terms at vertices from products of fields.



$$\int \dots \left(1 + \frac{\not{a}(n.x)\not{n}}{2n.p'}\right)\gamma^\mu\left(1 + \frac{\not{n}\not{a}(n.x)}{2n.p}\right)G_{\mu\nu}\dots$$

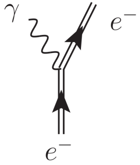
- Gives correction to the QED vertex:

$$\gamma^\mu \to \Gamma^\mu := \gamma^\mu + \frac{\delta m^2 n^\mu}{2n.(p+k)n.p}$$

The VSR vertex and the Ward Identity

☹ However . . .

- Extra slowly-varying terms at vertices from products of fields.



$$\int \ldots \left(1 + \frac{\not{a}(n.x)\not{n}}{2n.p'}\right)\gamma^\mu \left(1 + \frac{\not{n}\not{a}(n.x)}{2n.p}\right) G_{\mu\nu} \ldots$$

- Gives correction to the QED vertex:

$$\gamma^\mu \to \Gamma^\mu := \gamma^\mu + \frac{\delta m^2 n^\mu}{2n.(p+k)n.p}$$

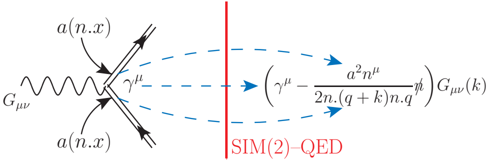
☺ The three-point vertex of VSR. ✓

Dunn & Mehen, hep-ph/0610202

The VSR vertex and the Ward Identity

- Provided all slowly varying terms are retained

Ilderton, PRD 94 (2016) 045019



1. VSR 3-point vertex recovered.
2. Ward Identity is preserved:

$$k.\Gamma = 0 \quad \checkmark$$

Higher-order vertices

☹ However... VSR has n-photon vertices for all n.

Dunn & Mehen, hep-ph/0610202

- Back to the propagator:

$$S_{\text{Volk}}(x,y) = \int \ldots \left(1 + \frac{\not{n}\not{a}(n.x)}{2n.p}\right) \ldots \not{p} \ldots \left(1 + \frac{\not{a}(n.y)\not{n}}{2n.p}\right)$$

- $n.x \neq n.y \implies a(n.x)a(n.y)$ rapidly oscillating.

Higher-order vertices

☹ However... VSR has n-photon vertices for all n.

Dunn & Mehen, hep-ph/0610202

- Back to the propagator:

$$S_{\text{Volk}}(x,y) = \int \ldots \left(1 + \frac{\not{n}\not{a}(n.x)}{2n.p}\right) \ldots \not{p} \ldots \left(1 + \frac{\not{a}(n.y)\not{n}}{2n.p}\right)$$

- $n.x \neq n.y \implies a(n.x)a(n.y)$ rapidly oscillating.
- What happens if $n.x = n.y$?

→ Short distance behaviour of the propagator.

Higher-order vertices

- Singular term in Volkov propagator:

$$S_{\mathsf{Volk}} \supset i\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\not{n}}{2n.p}e^{-ip.(x-y)} \;\propto\; \delta(n.x-n.y)\not{n}\;.$$

- 'Instantaneous lightfront propagator'.

Brodsky et al, Phys.Rept. 301 (1998), Bakker . . . A.I. et al, Nucl.Phys.Proc.Suppl 251 (2014) 165

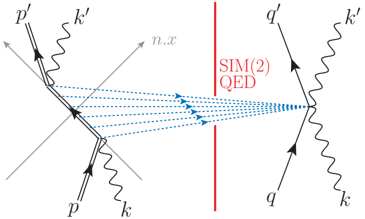
Higher-order vertices

- Singular term in Volkov propagator:

$$S_{\mathsf{Volk}} \supset i \int \frac{\mathrm{d}^4p}{(2\pi)^4} \frac{\not{n}}{2n.p} e^{-ip.(x-y)} \propto \delta(n.x - n.y)\not{n}\ .$$

- 'Instantaneous lightfront propagator'.

Brodsky et al, Phys.Rept. 301 (1998), Bakker . . . A.I. et al, Nucl.Phys.Proc.Suppl 251 (2014) 165



- Generates precisely the higher-order VSR vertices. ✓

Ilderton, PRD 94 (2016) 045019

VSR QED $\leftrightarrow$ SFQED

- VSR: an effective description of QED in a high frequency wave

- What is the VSR 'æther'?
- Preferred direction?
- Nonlocal. . .
- . . . but trans. invariant.

- Background EM wave.
- Wave vector.
- Interaction with the wave.
- Restored by averaging.

Conclusions

Summary

- SIM(2) QED as an effective theory of SFQED.
- New connections between
  - BSM physics with/without Lorentz violation.
  - SFQED and Lorentz invariance violation

Work in progress

- Other VSR groups?
- Other sectors of the standard model?
- Neutrino mass in this picture? (From e.g. loop effects.)
- Laser physics as "VSR in the lab"?

Heinzl and Ilderton, to appear